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*Erratum*

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## **Three-Valued Derived Logics for Classical Phase Spaces**

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The above paper from *International Journal of Theoretical Physics*, **35**, 31–62 (1996) contains an erroneous definition. Definition 4.3 (page 47) should be replaced by the following:

A proposition  $P$  is said to be *proper* if and only if there is no open set  $U$  such that  $P_0$  is dense in  $U$  but  $\text{int}(P_0 \cap U) = \emptyset$ , where  $P_0$  is the canonical representative of  $P$ .

Theorem 4.4 should then be restricted to sets which are elements of proper propositions.

The motivation for this corrected definition is the same as that for the original definition: the measurement status of everywhere dense sets is problematic. For example, in the 2 dimensional Cartesian coordinate plane, there is no finite collection of finite precision measurements which will distinguish between the set of points with rational x-coordinate and the set of points with irrational x-coordinate. It was originally thought that these problems could be avoided by excluding the proposition where the canonical representative was the entire space and, by duality, excluding the empty proposition. We have since realized that this reasoning is incorrect. In a sense, the measurement problem related to everywhere dense, empty interior sets is scale invariant. That is, in order to have distinguishable propositions, we must exclude those which are locally dense with empty interior; hence, the corrected definition.

It should be noted that there is a three-valued derived logic for phase spaces which, in general, is not equivalent to the ones described in the paper but it does avoid the notion of proper and improper propositions. The description of this logic is beyond the scope of the subject paper and certainly beyond the scope of this correction.